Mathematics and Physics of Topological Matter

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Keywords	Theoretical studies related to particle-, nuclear-, cosmic ray and astro-physics (15010)

Research Topics

- Topological insulator (Quantum Hall effect, etc.)
- Non-commutative geometry
- Topological aspects of quantum mechanics

Research Seeds

- Quantum Hall effect (topological insulator)

I am interested in the physics of topological insulators, particularly in quantum Hall effect. A quantum Hall system has bulk property of insulation, but its boundary property is metallic. Importantly, the metallic boundary current is immune to impurity scattering. Its robustness is guaranteed by topological invariant defined in the bulk.

I have been working on the relativistic quantum Hall effect in graphene and higher even-dimensional quantum Hall effect from a theoretical standpoint. Although the quantum Hall effect was originally formulated in two dimensions, we generalized it in higher even dimensions and unveiled universal formulation of the quantum Hall effect. In a modern perspective, the higher even-dimensional quantum Hall effect is classified to the topological insulator of A-class. Although the higher dimensional quantum Hall effect has been regarded merely as a hypothetical system, a modern cold atom system of optical lattice has provided an opportunity to simulate higher dimensional systems experimentally, regarding the energy spectrum as a new dimension. Future developments of higher dimensional physics are greatly anticipated.

I have also studied odd-dimensional generalization of the quantum Hall effect. Whereas a natural setup for the quantum Hall effect is even-dimensional, I have demonstrated that by regarding the odd-dimensional system as a subspace embedded in one dimension higher than an even-dimensional one, the odd-dimensional system finds its natural origin. I expect to clarify the physical properties of the odd dimensional, especially three-dimensional, quantum Hall effect in future studies. I am also interested in topological metals such as a Weyl semimetal – a three-dimensional counterpart of graphene.

- Non-commutative geometry

By dividing space-time into extremely fine pieces, it is expected that one can eventually encounter a quantum unit of space-time. Non-commutative geometry is a mathematical framework to describe the geometry comprising no-longer divisible units. I have been exploiting non-commutative geometry that effectively appears in the context of quantum Hall effect. I am particularly working on spherical non-commutative geometry realized in the magnetic field generated by a monopole. This system has a close relation to deep mathematical concepts such as the Hopf fibration and index theorem.

Related Technology

- Mathematica calculation
- Advanced mathematics such as group theory, differential geometry, and topology